arXiv:1209.0748v1 [cs.CG] 4 Sep 2012

Force-Directed Graph Drawing Using Social Gravity and Scaling

Michael J. Bannister David Eppstein
Michael T. Goodrich Lowell Trott

Department of Computer Science, University of California, Irvine

September 5, 2012

Abstract

Force-directed layout algorithms produce graph drawings by resolving a system of emulated physical forces. We present techniques for using *social gravity* as an additional force in force-directed layouts, together with a *scaling* technique, to produce drawings of trees and forests, as well as more complex social networks. Social gravity assigns mass to vertices in proportion to their network centrality, which allows vertices that are more graph-theoretically central to be visualized in physically central locations. Scaling varies the gravitational force throughout the simulation, and reduces crossings relative to unscaled gravity. In addition to providing this algorithmic framework, we apply our algorithms to social networks produced by Mark Lombardi, and we show how social gravity can be incorporated into force-directed Lombardi-style drawings.

1 Introduction

In a *social network*, vertices represent social entities, such as people or corporations, and edges represent relationships, such as friendship or partnership, between pairs of such entities. Online social networks, such as Facebook and Google+, are common illustrations of people as actors, and mutual friendship as edge relations [22]. The study of social networks, therefore, provides

a methodological approach to understanding social structure using graph-theoretic techniques (e.g., see [1]).

In the study of social networks, it is often important to distinguish between the importance or *centrality* of its vertices, or more accurately, of the entities they represent. Social networking researchers have identified several different centrality measures, each of which provides a way to relate the value of a social entity to the structure of the network to which it belongs. We consider three of the most commonly-used types of centrality in this paper: *degree centrality*, *betweenness centrality*, and *closeness centrality* (e.g., see [18]). The degree centrality of vertex v is defined simply to be its degree. The closeness centrality of a vertex v is defined to be the reciprocal of the mean distance from v to all other vertices in the network. The betweenness centrality of a vertex v is defined to be the proportion of paths containing v among all shortest paths in the network, which can be expressed as the sum

$$\sum_{s=t=v} {}_{st}(v)/\ {}_{st},$$

where ${}_{st}$ is the number of shortest paths from s to t, and ${}_{st}(v)$ is the number of shortest paths from s to t that contain v. For more information on centrality and its efficient computation, see [3, 32].

Naturally, the drawing of social networks is an important part of social science research (e.g., see Brandes *et al.* [5]). Since centrality is a common focus of social network analysis, we feel that placing vertices with significant centrality close to the middle of a drawing should be a natural goal of drawing algorithms for social networks, and the approach we explore in this paper for achieving this goal is based on the framework of force-directed layouts.

Force-directed layout algorithms are well-known in the graph drawing literature, as they yield reasonable drawings for a wide variety of graphs (e.g., see [2, 4, 13, 21, 23, 24]). Traditionally, these methods use a graph's structure to mimic a physical system of attractive spring forces along edges and universal repulsive forces emanating from vertices. The system of resulting force equations is then resolved into a minimal-energy state, which produces a drawing.

Frick *et al.* [20] introduce the use of gravitational forces in such layout methods in addition to spring and repulsive forces, and similar gravitational forces have been utilized in several other force-directed systems as well (e.g., see [17, 31]). The forces used in these systems are intended to pull vertices

towards the center of a drawing using a "mass" that is proportional to vertex degree (that is, using something akin to degree centrality), and, in so doing, to speed up algorithmic convergence. In this paper, we study force-directed methods based on the use of a generalized gravitational force, which we call *social gravity*, where the mass of a vertex can depend on any social-networking centrality measure.

1.1 Related Work

Brandes *et al.* [6] describe a network visualization tool that places vertices on concentric rings whose radii are based on centrality measures. Correa *et al.* [11] perform a study based on visual reasoning about social networks and about how centrality can be used as an aid in network simplification, including use of a tool that places vertices on concentric rings in a manner similar that that of Brandes *et al.* [6]. Brandes and Pich [8] show how to maintain the approach of such radial layouts while using stress minimization. These works differ from our use of social gravity in that radial drawings constrain vertices to discrete rings based on centrality values, whereas force-directed placement using social gravity is more continuous. For additional work on other techniques for visualizing social networks, please see, e.g., [6, 7, 9, 19, 26].

A prime example of culturally relevant social network drawing is the work of Mark Lombardi [27]. Mark Lombardi was an American artist who created drawings attempting to document financial and political conspiracies. His drawings are recognizable for the use of circular arcs for edges and for their aesthetically pleasing placement of vertices and edges. Lombardi's art has inspired algorithmic methods in graph drawing (e.g., see [14–16]), combining the ideas of drawing edges as circular arcs and of optimizing the angular resolution of the vertices. Most relevantly for our work, Chernobelskiy *et al.* [10] introduce a force-directed approach to produce Lombardi-style graph drawings with perfect or near-perfect angular resolution and circular-arc edges. They do not consider gravitational forces in their methods, however.

1.2 Our Results

As mentioned above, force-directed algorithms are an efficient and effective approach to graph drawing. In this paper, we study methods for harnessing *social gravity* to control the positioning of vertices in a force-directed layout.

That is, we show how to incorporate gravitational forces into a force-directed layout so that vertices are affected by gravity in relation to various social-networking centrality measures.

As it is in the physical world, gravity can be a powerful force in graph drawing, which can overwhelm other forces. In force-directed algorithms, strong gravitational forces can cause a drawing to "freeze" into a local minimum configuration with many crossings. To ameliorate this effect, we use gravity in conjunction with a *scaling* technique: we perform a force-directed simulation that starts without gravity, so as to untangle edges and reduce crossings, and we then gradually increase the influence of gravity so as to push vertices to the center in proportion to their importance. In contrast, previous force-directed methods using gravity [20] have a parameter similar to our variable scaling factor, $_t$, but set it to be a fixed constant.

Using social gravity and scaling allows us to produce drawings that have all the classic advantages of force-directed algorithms, such as being space efficient and visually pleasing, while also centralizing socially significant vertices. We examine the social networks of Mark Lombardi as a test case that shows that combining our ideas of social gravity and scaling in conjunction with Lombardi-style force-directed drawing [10] can create effective graph drawings for social networks.

2 The Algorithm

Our algorithm has a structure common to most force-directed algorithms. For the forces considered, we attempt to place vertices of the graph such that they are in equilibrium, i.e., so that the net force on each vertex is zero. To achieve this goal we compute the net force on each vertex, and then move the vertex in the direction of this net force. Convergence is sped up by normalizing the net force down. This two-step process is then repeated until the vertices are approximately in equilibrium or the maximum number of iterations has been reached.

The forces we consider in our algorithm are the attractive and repulsive forces of Fruchterman and Reingold [21],

$$f_r(u,v) = \frac{k^2}{P[u] - P[v]^{\ 2}}(P[v] - P[u]) \quad f_a(u,v) = \frac{P[u] - P[v]}{k}(P[u] - P[v])$$

where $P[v]$ is the current position of vertex v, and k is a parameter controlling

the *natural length* of an edge so that, e.g., when drawing K_2 its edge will have length k. We then augment these forces with a gravitational force

$$f_g(v) = \ {}_t M[v](\ - P[v])$$

where $M[v]$ is the mass of vertex v, $\quad = \quad {}_v P[v]/|V|$ is the centroid of the points, and ${}_t$ is the gravitational scaling parameter that varies per iteration. This gravitational force is inspired by the work of Frick *et al.* [20]. However, as mentioned above, we consider mass based on social networking parameters as opposed to just degree. In addition, we consider the gravitational force primarily as a force for shaping the drawing rather than as a tool to speed up convergence, so we use larger values for ${}_t$ than Frick *et al.*

To reduce crossings in the final drawing, our *scaling* technique gradually increases the value for ${}_t$ through the iterations. For the drawings in this paper, we control this increase using a step function, where ${}_t$ starts at 0 and increases by approximately 0.2 when the vertices reach an approximate equilibrium. How high ${}_t$ should go may depend on the graph, but a maximum value of 2.5 seems to yield pleasing results.

2.1 Pseudo-code

Our implementation starts by precomputing the mass of each vertex and storing these values in a dictionary M. The computation of M varies depending on which form of centrality is being used. After this initialization we run the Algorithm 1, shown below.

Algorithm 1 Force-directed layout with social gravity and scaling

for t 1 to `max_iteration` **do**
  ${}_t$ according to a scaling schedule
  for each vertex v in graph G **do**
    $I[v] \quad f_r(u,v) + \quad f_a(u,v) + \quad f_g(v)$
    $u=v \quad \{u,v\} \ E \quad v$
  end for
  for each vertex v in graph G **do**
    $P[v] \quad P[v] + \ \cdot \min(I_{\max}, I[v])$
  end for
end for

Before each iteration, the gravitational parameter, $_t$, is updated according to a scaling schedule. After the impulse force is computed for each vertex, it is capped to $I_{\max}$ and scaled by . In our runs, we typically set $k = 80$, $I_{\max} = 10$, $= 0.1$, $_t = 0.2\ t/200$, and `max_iteration` is set so that $_t$ 2.5. Example applications of our algorithm are given in Sections 3 and 4.

3 Trees and Forests

As a first demonstration of our algorithm, we show how it performs on trees and forests. The algorithm takes as input an unrooted tree, and makes no explicit attempt to identify a root, but our drawings can be viewed as being rooted at a node near the center of the drawing, which often tends to be one of high degree, closeness, or betweenness centrality. In Figure 1, we see that our algorithm (bottom row) places the vertices of the graph uniformly in a disk, whereas the standard force-directed algorithms place the graph in a more elongated region. In addition to better space utilization, our algorithm tends to have better angular resolution than existing force directed algorithms, as can also be seen in Figure 1.

An added benefit of the social-gravity force is that it allows a force-directed algorithm to draw trees more compactly, which improves the area bound for a drawing and allows for individual vertices to be drawn larger than they would be in the more widely-spaced drawings produced by traditional force-directed methods. This effect becomes even more noticeable as the size of the graph increases, as can be see in Figure 2. This compaction is done without introducing clutter or many crossings, due to our use of scaling to initially emphasize the repulsive force that applies to vertices. The compacting effect of social gravity is especially helpful for drawings of disconnected forests, since it keeps the trees that form the forest from drifting apart, as shown in Figure 3.

In each case, the use of social gravity and scaling implies that the resulting drawing tends to have vertices placed uniformly in a disk with few if any crossings. In addition, the algorithm tends to place the larger trees in a forest near the center of the disk. (See Figures 4 and 5.)

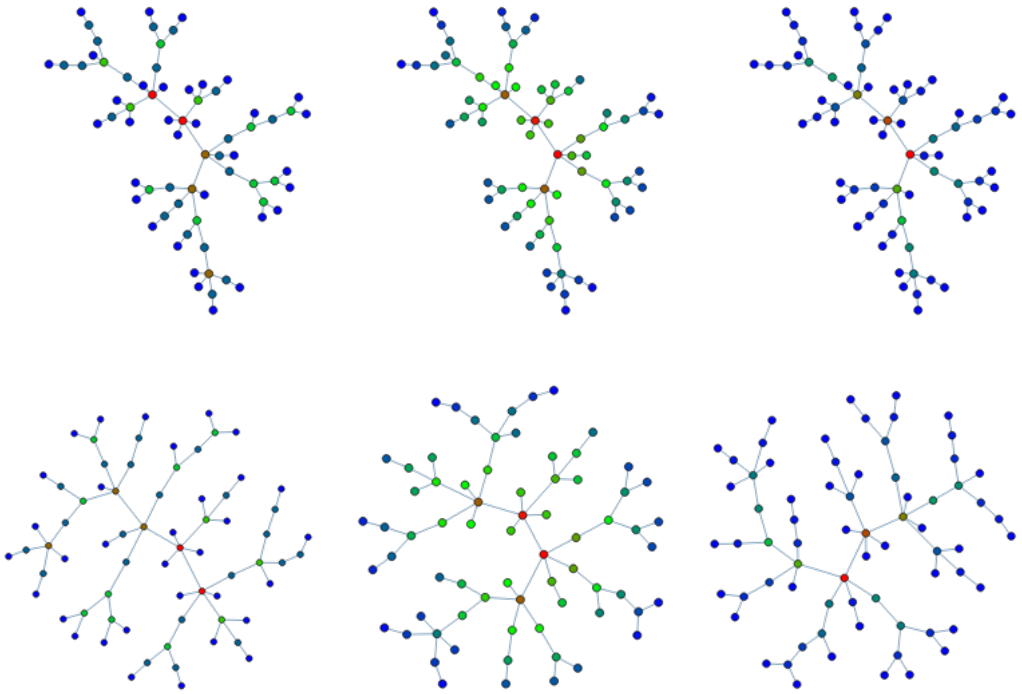

Figure 1: A tree of 70 vertices. The top row is the classic force-directed embedding colored from left to right with degree, closeness, and betweenness centrality respectively. The bottom row has the corresponding gravitational force-directed embedding.

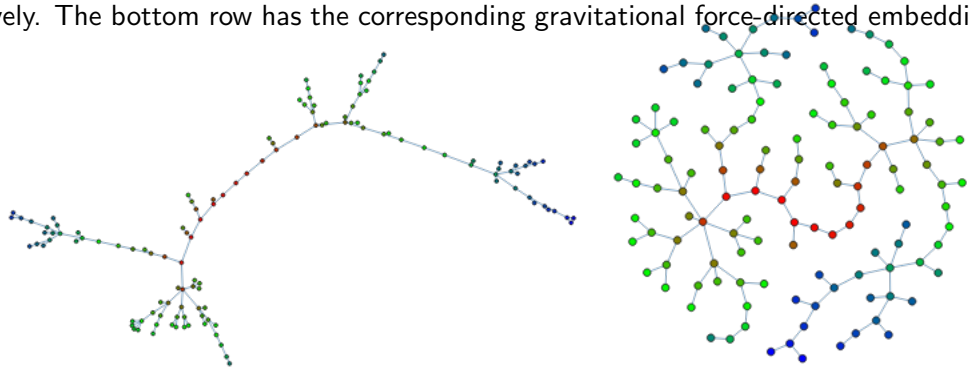

Figure 2: A tree with 126 vertices drawn on the left with the classical force based algorithm, and drawing with scaled gravity on the right.

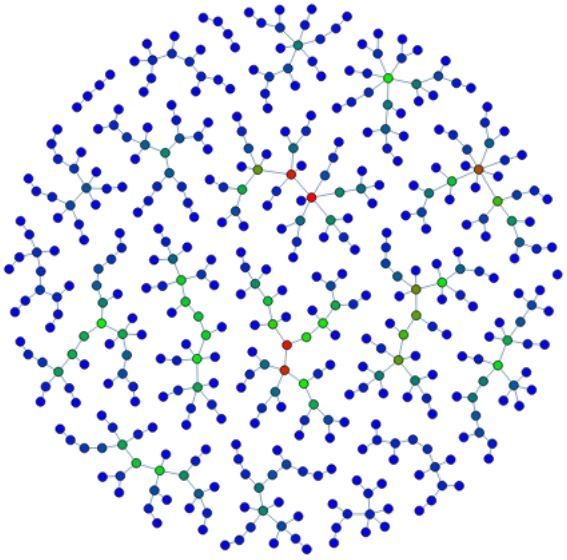

Figure 3: A forest with 422 vertices and 20 trees, drawn with gravity based on betweenness centrality.

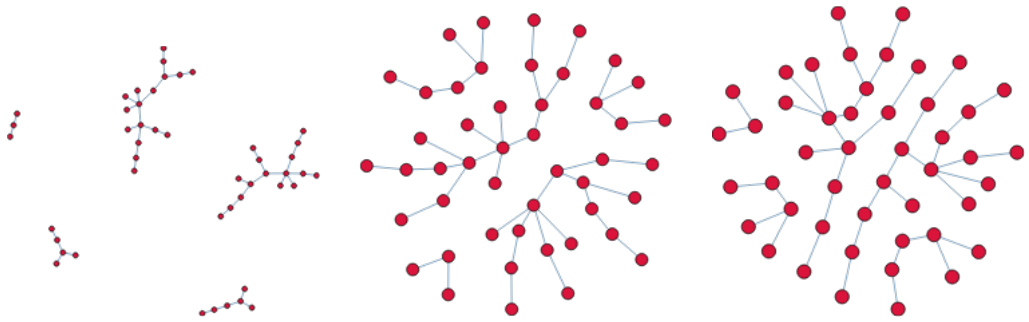

Figure 4: A five component forest with 45 vertices drawn without and with a gravitational force. The figure on the left has no gravity, in the middle gravity is scaled up, while the figure on the right has uniformly strong gravity throughout the algorithm.

3.1 The Impact of Scaling

The impact of the scaling of social-gravity forces on vertices over the course of the algorithm becomes important for the ultimate placement of vertices in a quality drawing. For instance, examine the five component forest in the far left of Figure 4. These trees have been drawn using a traditional force-directed algorithm. Components have each achieved a low energy state,

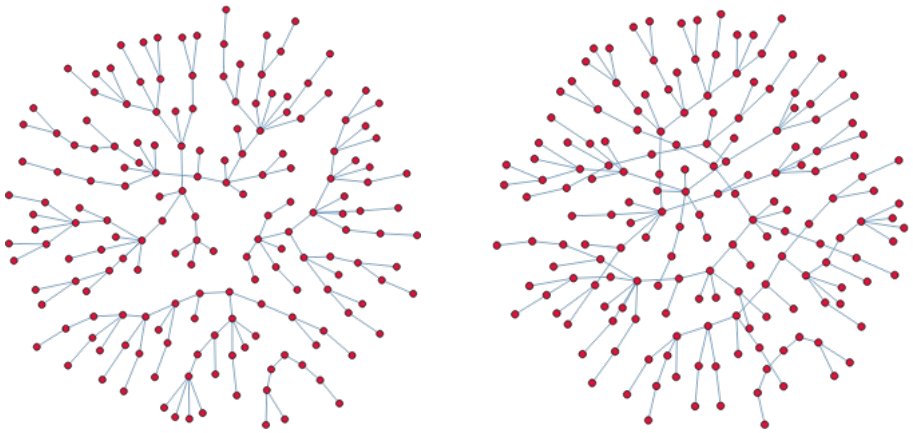

Figure 5: A five component forest with 174 vertices with a gravitational force. The figure on the left has gravity scaled up, while the figure on the right does not.

and then begun to spread apart via repulsive forces. The same forest is then redrawn with gravity using two approaches. In the drawing in the center of Figure 4, gravity is applied in increasing amounts, beginning with no gravity, and increasing according to a step function. In the drawing on the right, gravity is applied at a uniformly high level throughout the drawing. In drawings with low number of vertices, the quality of the drawing is robust to the method of gravity application. If we examine the larger forest in Figure 5, for example, with the left drawing having scaled gravity and the right drawing having uniformly high gravity, we see that scaling gravity dramatically reduces crossings in the final output. Our findings indicate that the benefits of using social gravity and scaling are achieved as long as the algorithm begins with weak or no gravity, and increases, even with few steps, to the maximum gravitational force.

3.2 The Impact of Using Different Forms of Centrality

While the various forms of centrality represent related ideas, the variation in definition means that gravitational forces based on these distinct forms will produce differing results. This can be illustrated with high degree vertices that occur near edge of the graph in a classic force-embedding. These vertices will have high degree centrality, but because of their exterior structural location, they will often have low closeness and betweenness centrality. In

degree centrality gravitational embedding, these vertices will be attracted to the center with more force, while embeddings with other centralities will leave them on the exterior of the graph.

In Figure 1 we see a comparison of the effects of gravity defined using different centralities on a tree of 70 vertices. The top row is the tree embedded with the force-directed method with no gravitational force, and colored from left to right with degree, closeness, and betweenness centrality. Red coloring represents vertices of high centrality, blue vertices have low centrality, and vertices in between are colored with a spectrum from red to blue according to their relative centrality in the graph. The lower row consists of the results of the gravitational force-directed algorithm where the mass relates the centrality of the vertices, once again, from left to right with degree, closeness, and betweenness centrality.

4 Social Networks

As our algorithm was motivated by the visualization of social networks, we chose example graphs in this application area as test inputs for it. Figure 6 shows several such graphs and their corresponding gravitational drawings. As the gravitational forces are exerted on vertices with high centrality, these are pulled towards the center of the drawing. This makes vertices that are more central to the social structure of the network visually localized. Prominent social forces are thus more easily identified, increasing discovery and communication of these social actors.

The overall effects of our algorithm on these graph drawings are similar to its effects on trees. The drawings produced by our algorithm tend to be more space efficient, as observed above. In the classic force directed model, long components that are distant from the most central vertices of the graph are repulsed by many other vertices, with no compensatory attractive forces, and tend to be stretched out from the center, causing the drawing to have a large bounding box. Since the gravitational force of our algorithm attracts these groupings of vertices, they are brought closer to the core of the graph, and are able to repulse each other, increasing visibility. This can been seen especially in the degree and betweenness centrality embeddings in Figure 6.

A less obvious trait of the gravitational force is that our drawings tend to exhibit is superior angular resolution than their classic force-directed counterparts. Vertices are distributed more uniformly as gravitational forces

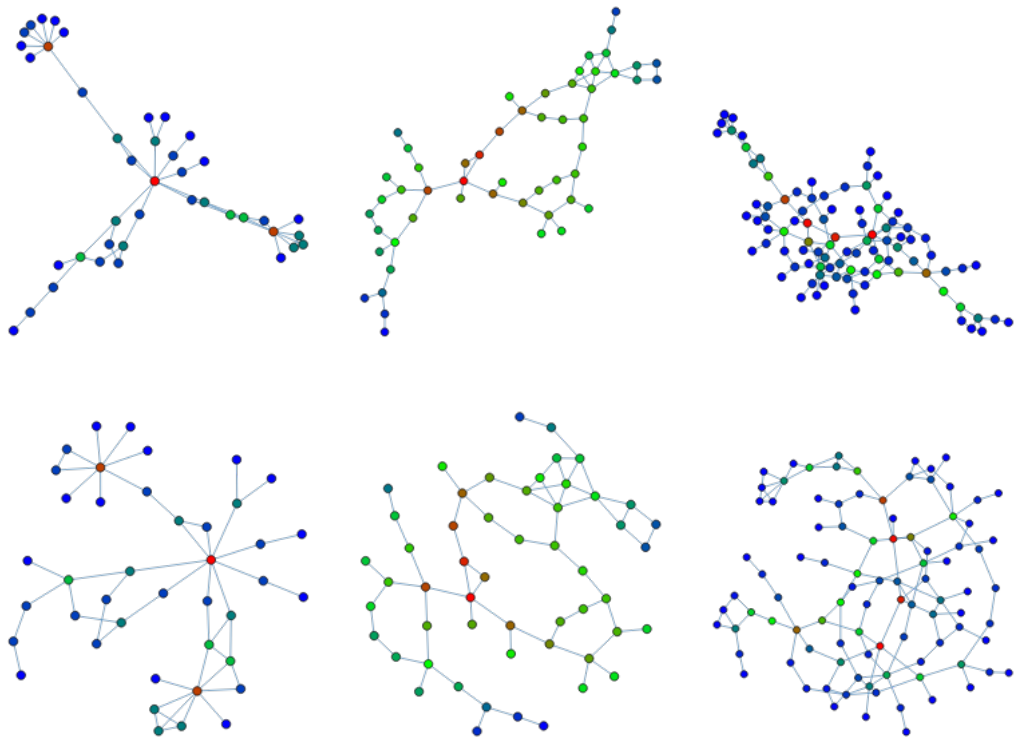

Figure 6: Comparison of graphs embedded with the classic force-directed algorithm(top row) and the gravitational force-directed algorithm(bottom row). The columns from left to right are colored and utilize the gravitational force on degree, closeness, and betweenness centrality respectively.

increase, and so adjacent vertices tend away from co-linearity. This phenomenon is clearly visible in the upper-left corner of the degree centrality drawings of Figure 6.

As with all force-directed drawing algorithms, finding the appropriate balance of the competing forces is important. Our implementation, while certainly using a gravitational force that was strong enough to effect the final outcome of the algorithm, did not make it overpowering in comparison with the classic forces. This was done to preserve all the benefits exhibited by the classical approach, while trying to enhance very successful drawings in tangible ways. Obviously, if some characteristic is especially desired, like the central movement of vertices with high centrality, then one can modify

the related forces accordingly. We demonstrate our algorithm on several additional examples of social networks in an appendix.

4.1 Lombardi-style Graph Drawing with Social Gravity

We also can apply our approach to Lombardi's social networks and Lombardi-style graph drawing. Specifically, in Figures 7 and 8 we have drawn two of the conspiracy networks documented by Mark Lombardi. In these graphs vertices represent people or business entities and edges represent business dealings. We illustrate the drawings of these networks with our gravitational force-directed algorithm. In Figure 8 we show our method in conjunction with the force-directed approach to Lombardi-style drawing [10], which uses circular-arc edges.

In the left panel of Figure 7 we see Lombardi's "Bill Clinton, the Lippo Group, and China Ocean Shipping" graph. This graph highlights the possible associations between illegal arms dealing by Chinese nationalists in Los Angeles and supposed White House campaign-finance corruption [27]. The middle panel is a straight line approximation of Lombardi's drawing colored with degree centrality. Note that many of the high degree vertices are condensed in the lower left section of the drawing. It is also clear that the nodes are packed much more densely in this region. If we examine the right panel, we see that the high degree vertices, plausibly the actors more central to the presented power relationship, are now more centralized. As we would expect, vertices also have a much more uniform spacial distribution. Note the increase in angular resolution of the central high centrality vertex, and the vertex in the upper left of the central drawing.

The left of Figure 8 contains another of Lombardi's drawings,"World Finance Corporation, Miami, Florida". This network represents the plausible role played by WFC in trafficking Colombian drugs and money laundering of the profits. The central drawing is the network embedded with betweenness centrality gravitational forces. As is desirable, the vertex of highest centrality has been centered in the output. The right drawing is the result of the application of the dummy-vertex approach to force-directed Lombardi-style drawing to the result of the our gravitational algorithm. While not as consistent as Lombardi's hand drawing, this automated approach does succeed in replicating some of the long arcs through multiple nodes found in the

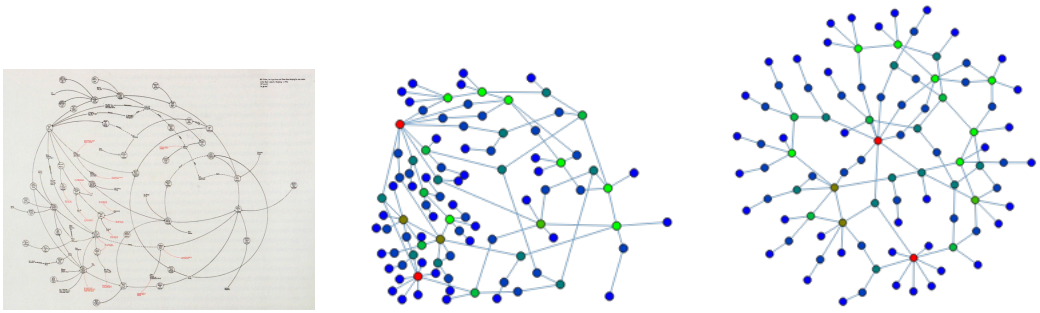

Figure 7: Lombardi's "Bill Clinton, the Lippo Group, and China Ocean Shipping" network with straight-line approximation and degree centrality gravitational embedding.

original. The creation of circular arc edges is done with classical forces, and thus augmenting the implementation of the gravitational algorithm to create Lombardi-style drawings is very simple.

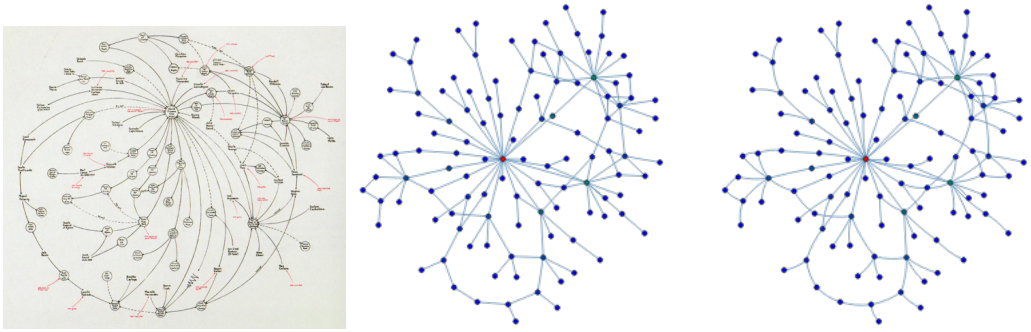

Figure 8: Lombardi's "World Finance Corporation, Miami, Florida." From left to right: Lombardi's drawing, the betweenness centrality gravitational drawing, and the central drawing with the addition of force-directed Lombardi-style edges.

5 Conclusions and Directions for Future Work

The addition of social gravity to force-directed drawings produces several positive results. Vertices with high centrality become centrally located, vertices

are distributed more uniformly in the drawing, and drawn graphs exhibit superior angular resolution. Moreover, scaling gravity has proven to be important to ensuring these positive traits. In particular, the benefits of social gravity and scaling lead to drawings of trees and forests that are aesthetically pleasing, compact, tend to be crossing free, and have uniform edge length. Social networks also exhibit these positive traits, although in some cases, this produces crossings that would have been otherwise avoided.

There are several directions for future work. Study of the ideal balance between gravitational forces and classical forces is left to future work. We have also shown that the force-directed approach to Lombardi-style drawing can be used in conjunction with our algorithm. We recommend future study of how the Lombardi forces should interact with the differentiated final edge length created by the output of the gravitational algorithm. Finally, there is also room for improvement in the runtime of our algorithm, which is quadratic as implemented, using methods from n-body simulation [25].

Acknowledgements

The work on this paper was supported in part by the U.S. Office of Naval Research, under MURI grant N00014-08-1-1015, and the U.S. National Science Foundation, under grants 0830403, 0953071, and 1011840.

References

[1] S. P. Borgatti, A. Mehra, D. J. Brass, and G. Labianca. Network Analysis in the Social Sciences. *Science* 323(5916):892–895, 2009, doi:10.1126/science.1165821.

[2] F. Brandenburg, M. Himsolt, and C. Rohrer. An experimental comparison of force-directed and randomized graph drawing algorithms. *3rd Symp. on Graph Drawing*, pp. 76–87. Springer, LNCS 1027, 1996, doi:10.1007/BFb0021792.

[3] U. Brandes. A faster algorithm for betweenness centrality. *The Journal of Mathematical Sociology* 25(2):163–177, 2001, doi:10.1080/0022250X.2001.9990249.

[4] U. Brandes. Drawing on Physical Analogies. *Drawing Graphs*, pp. 71–86. Springer, LNCS 2025, 2001.

[5] U. Brandes, L. C. Freeman, and D. Wagner. Social networks. *Handbook of Graph Drawing and Visualization*, pp. 26-1–26-32. Chapman & Hall/CRC, 2013.

[6] U. Brandes, P. Kenis, and J. Raab. Explanation Through Network Visualization. *Methodology: European Journal of Research Methods for the Behavioral and Social Sciences* 2(1):16–23, 2006, doi:10.1027/1614-2241.2.1.16.

[7] U. Brandes, P. Kenis, J. Raab, V. Schneider, and D. Wagner. Explorations into the Visualization of Policy Networks. *Journal of Theoretical Politics* 11(1):75–106, 1999, doi:10.1177/0951692899011001004.

[8] U. Brandes and C. Pich. More Flexible Radial Layout. *Journal of Graph Algorithms and Applications* 15(1):157–173, 2011, `http://jgaa.info/accepted/2011/BrandesPich2011.15.1.pdf`.

[9] P. J. Carrington, J. Scott, and S. Wasserman. *Models and Methods in Social Network Analysis*. Cambridge University Press, 2005, p. 248ÔÇô269. Graphic techniques for exploring social network data. by Linton C. Freeman.

[10] R. Chernobelskiy, K. Cunningham, M. T. Goodrich, S. Kobourov, and L. Trott. Force-directed Lombardi-style graph drawing. *Graph Drawing*, pp. 320–331. Springer, LNCS 7034, 2012, `http://dx.doi.org/10.1007/978-3-642-25878-7_31`.

[11] C. Correa, T. Crnovrsanin, and K.-L. Ma. Visual reasoning about social networks using centrality sensitivity. *IEEE Trans. on Visualization and Computer Graphics* 18(1):106–120, jan. 2012, doi:10.1109/TVCG.2010.260.

[12] P. De, A. E. Singh, T. Wong, W. Yacoub, and A. M. Jolly. Sexual network analysis of a gonorrhoea outbreak. *Sexually transmitted infections* 80(4):280–5, August 2004, doi:10.1136/sti.2003.007187.

[13] G. Di Battista, P. Eades, R. Tamassia, and I. G. Tollis. *Graph Drawing: Algorithms for the Visualization of Graphs*. Prentice Hall PTR, Upper Saddle River, NJ, USA, 1998.

[14] C. Duncan, D. Eppstein, M. T. Goodrich, S. Kobourov, and M. Löffler. Planar and poly-arc Lombardi drawings. *Graph Drawing*, pp. 308–319. Springer, LNCS 7034, 2012, `http://dx.doi.org/10.1007/978-3-642-25878-7_30`.

[15] C. A. Duncan, D. Eppstein, M. T. Goodrich, S. G. Kobourov, and M. Nöllenburg. Drawing Trees with Perfect Angular Resolution and Polynomial Area. *18th Symp. on Graph Drawing*, pp. 183–194, 2010.

[16] C. A. Duncan, D. Eppstein, M. T. Goodrich, S. G. Kobourov, and M. Nöllenburg. Lombardi drawings of graphs. *18th Symp. on Graph Drawing*, pp. 195–207, 2010.

[17] T. Dwyer and P. Eades. Visualising a fund manager flow graph with columns and worms. *6th Int. Conf. on Information Visualisation*, pp. 147–152, 2002, doi:10.1109/IV.2002.1028770.

[18] L. C. Freeman. Centrality in social networks conceptual clarification. *Social Networks* 1(3):215–239, 1979, doi:10.1016/0378-8733(78)90021-7.

[19] L. C. Freeman. Visualizing social networks. *Journal of Social Structure* 1(1), 2000.

[20] A. Frick, A. Ludwig, and H. Mehldau. A Fast Adaptive Layout Algorithm for Undirected Graphs. *Graph Drawing*, pp. 388–403. Springer Berlin / Heidelberg, Lecture Notes in Computer Science 1984, 1994.

[21] T. M. J. Fruchterman and E. M. Reingold. Graph drawing by force-directed placement. *Software: Practice and Experience* 21(11):1129–1164, 1991, doi:10.1002/spe.4380211102.

[22] B. Furht. *Handbook of Social Network Technologies and Applications*. Springer-Verlag, 2010.

[23] P. Gajer, M. T. Goodrich, and S. G. Kobourov. A multi-dimensional approach to force-directed layouts of large graphs. *Comp. Geometry: Theory and Applications* 29(1):3–18, 2004.

[24] P. Gajer and S. Kobourov. GRIP: Graph Drawing with Intelligent Placement. *Graph Drawing*, pp. 104–109. Springer, LNCS 1984, 2001.

[25] Y. F. Hu. Efficient and high quality force-directed graph drawing. *The Mathematica Journal* 10(1):37–71, 2005.

[26] A. S. Klovdahl. A note on images of networks. *Social Networks* 3:197–ÔÇô214, 1981.

[27] M. Lombardi and R. Hobbs. *Mark Lombardi: Global Networks*. Independent Curators, 2003.

[28] H. Newman and J. O. Newman. *A Genealogical Chart of Greek Mythology*. The University of North Carolina Press, 2007.

[29] J. J. Potterat, S. Q. Muth, R. B. Rothenberg, H. Zimmerman-Rogers, D. L. Green, J. E. Taylor, M. S. Bonney, and H. A. White. Sexual network structure as an indicator of epidemic phase. *Sexually transmitted infections* 78 Suppl 1:i152–8, April 2002.

[30] J. J. Potterat, L. Phillips-Plummer, S. Q. Muth, R. B. Rothenberg, D. E. Woodhouse, T. S. Maldonado-Long, H. P. Zimmerman, and J. B. Muth. Risk network structure in the early epidemic phase of HIV transmission in Colorado Springs. *Sexually transmitted infections* 78 Suppl 1:i159–63, April 2002.

[31] G. Sander. Graph layout for applications in compiler construction. *Theoretical Computer Science* 217(2):175–214, 1999, doi:10.1016/S0304-3975(98)00270-9, `http://www.sciencedirect.com/science/article/pii/S0304397598002709`.

[32] K. Stephenson and M. Zelen. Rethinking centrality: Methods and examples. *Social Networks* 11(1):1–37, 1989, doi:10.1016/0378-8733(89)90016-6.

A Additional Drawings of Social Networks

We demonstrate our algorithm on some additional types of social networks in this appendix. In Figure 9 we draw a graph of genealogy of the Greek gods and related characters. The edges in this graph are drawn from parent to child. In an another type of social network, we draw sexual networks collected by voluntary surveys during sexually transmitted infection outbreaks in Figure 10.

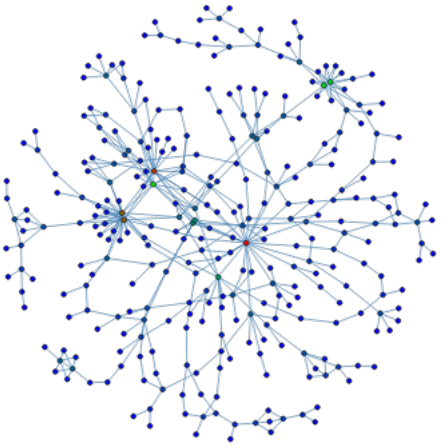

Figure 9: A genealogy graph of the Greek gods [28].

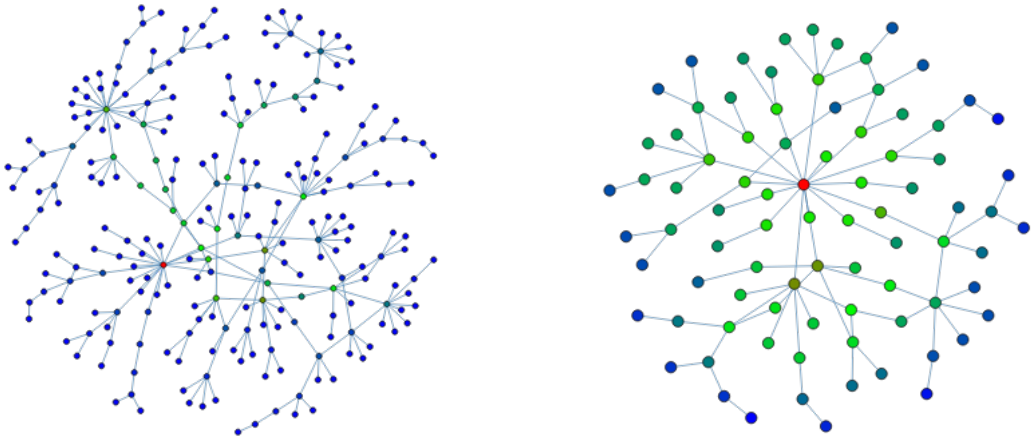

Figure 10: Sexual networks corresponding on the right to an HIV outbreak [30], and a gonorrhea outbreak on the left [12, 29].